# Math 2551 A1-3 Final

**Section:**
**Name:**
**Student ID:**

(1) Solve the initial value problem for $\mathbf{r}$ as a vector function of $t$.

$$\frac{d^2\mathbf{r}}{dt^2} = -32\mathbf{j},$$

$$\mathbf{r}(0) = 2\mathbf{j} \text{ and } \frac{d\mathbf{r}}{dt}|_{t=0} = 50\sqrt{2}\mathbf{i} + 50\sqrt{2}\mathbf{j}.$$

(2) Find $\mathbf{T}, \mathbf{N}$ and curvature $k$ of the curve

$$\mathbf{r}(t) = (\cos t + t \sin t)\mathbf{i} + (\sin t - t \cos t)\mathbf{j} + 3\mathbf{k}, \quad t > 0.$$

(3) (a) Find $\partial z/\partial y$ at the point $(x, y, z) = (0, \pi, 0)$ assuming that the equation $\sin(2x+y)+\sin(y+3z)+\sin(x+z) = 0$ defines $z$ as a differentiable function of $x$ and $y$ near the point.

$$(b)\ \ F(x) = \int_0^{x^4} \sqrt{t^4 + x^3}dt, \ \text{ find } \frac{dF}{dx}.$$

(c) Find the linearization $L(x, y, x)$ of $f(x, y, z) = e^x + \sin(y + z)$ at $(0, \frac{\pi}{2}, 0)$.

(4) Suppose that the Celsius temperature at the point $(x, y, z)$ on the sphere $x^2 + y^2 + z^2 = 1$ is $T = 400xyz^2$. Locate the highest and lowest temperatures on the sphere.

(5) Let $D$ be the smaller cap cut from a solid ball of radius 2 units by a plane $\sqrt{2}$ units from the center of the sphere. Express the volume of $D$ as an iterated triple integral in (a) spherical, (b) cylindrical, and (c) rectangular coordinates.

(6) Determine whether the vector field $\mathbf{F}$ is conservative or not, and find a potential function $f$ for $\mathbf{F}$ if it's conservative.

$$\mathbf{F} = (y \sin z)\mathbf{i} + (x \sin z)\mathbf{j} + (xy \cos z)\mathbf{k}$$

(7) (a) Find the counterclockwise circulation for the field $\mathbf{F} = (x + y)\mathbf{i} - (x^2 + y^2)\mathbf{j}$ along the curve $C$: The triangle bounded by $y = 0$, $x = 1$, and $y = x$.

(b) Find the area of the region enclosed by the curve $C_1 : \mathbf{r}(t) = (2 + \cos t)\mathbf{i} + (2 + \sin^3 t)\mathbf{j}$, $0 \leq t \leq 2\pi$.

(8) Let $S$ be the portion of the cylinder $y = \ln x$ in the first octant whose projection parallel to the $y$-axis onto the $xz$-plane is the rectangle $R_{xz}: \ 1 \leq x \leq e, \ 0 \leq z \leq 1$. Let $\mathbf{n}$ be the unit vector normal to $S$ that points away from the $xz$-plane. Find the flux of $\mathbf{F} = 2y\mathbf{j} + z\mathbf{k}$ through $S$ in the direction of $\mathbf{n}$: $\int\int_S \mathbf{F} \cdot \mathbf{n} d\sigma$.