Math 2551 A1-3 Final Total: 40 points

**Section:**
**Name:**
**Student Number:**

**For problems (1)-(4), fill in the blanks.** (One point for each blank).

(1) $\mathbf{r}(t) = \frac{\sqrt{2}}{2}t\mathbf{i} + (\frac{\sqrt{2}}{2}t - 16t^2)\mathbf{j}$ is the position of a particle at time t. The angle between the velocity and acceleration vectors at the time $t = 0$ is ( ).

(2) Given that $\frac{d^2\mathbf{r}}{dt^2} = -(\mathbf{i}+\mathbf{j}+\mathbf{k})$ and $\mathbf{r}(0) = 10\mathbf{i}+10\mathbf{j}+10\mathbf{k}$ and $\frac{d\mathbf{r}}{dt}|_{t=0} = 0$, $\mathbf{r}(t) =$ ( ).

(3) The length of the curve $\mathbf{r}(t) = \sqrt{2}t\mathbf{i} + \sqrt{2}t\mathbf{j} + (1-t^2)\mathbf{k}$ from $(0,0,1)$ to $(\sqrt{2}, \sqrt{2}, 0)$ is ( ). (There is no need to evaluate the definite integral).

(4) $\mathbf{r}(t) = 6\sin(2t)\mathbf{i} + 6\cos(2t)\mathbf{j} + 5t\mathbf{k}$, then $\mathbf{T} =$ ( ), $\mathbf{N} =$ ( ), and the curvature $k =$ ( ).

**For problems (5)-(31), mark "true" or "false" for each statment.** (One point for each statement).

(5) If the acceleration of a particle is always orthogonal to its velocity, then the speed of the particle is a constant.

(6) A normal vector of the level surface of

$$g(x,y,z) = \int_x^y \frac{ydt}{1+t^2} + \int_0^z \frac{d\theta}{\sqrt{4-\theta^2}}$$

at $(0, 1, \sqrt{3})$ is $(-1, 1/2, 1)$.

(7)

$$\lim_{(x,y)\to(1,\pi/6)} \frac{x\sin y}{x^2+1} = 1/4$$

(8)

$$\lim_{(x,y)\to(0,0)} x\cos(1/y) = 0$$

(9)

$$\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2} = 1$$

(10) Let $A(c,h,k,m,q) = \frac{km}{q} + cm + \frac{hq}{2}$, then $\frac{\partial A}{\partial q} = -\frac{km}{q^2} + cm + \frac{h}{2}$.

Let

$$f(x,y) = \begin{cases} 0, & x^2 < y < 2x^2 \\ 1, & \text{otherwise.} \end{cases}$$

Then (11) $f_x(0,0) = 1$.
(12) $f$ is differentiable at $(0,0)$.

(13) Let $w = 2ye^x - \ln z$, $x = \ln(t^2+1)$, $y = \cos t$, and $z = e^t$, then $\frac{dw}{dt}|_{t=0} = -1$

(14) $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} - 1 = 0$. Then $\frac{\partial z}{\partial x}|_{(2,3,6)} = -9$.

(15) Let $F(x) = \int_{x^2}^{1} \sqrt{t^3+x^2}\,dt$, then $F'(1) = -2\sqrt{2}$.

(16) The derivative of $h(x,y,z) = \cos(xy) + e^{yz} + \ln(zx)$ at the point $(1,0,1/2)$ in the direction of $\mathbf{u} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ is 8.

(17) Let $f(x,y) = \frac{x-y}{x+y}$. The unit vector for which $f'_{\mathbf{u}}(-\frac{1}{2}, \frac{3}{2})$ is smallest is $\mathbf{u} = -\frac{1}{\sqrt{10}}(3,1)$.

(18) An equation for the plane that is tangent to the surface $z = \sqrt{y-x}$ at $(1,2,1)$ is $(x-1) - (y-2) - 2(z-1) = 0$.

(19) The quadratic approximation of $f(x,y) = \sin x \cos y$ at the origin by Taylor's formula is $x$.

$$(20)\quad \int_0^3 \int_{\sqrt{x/3}}^1 e^{y^3}\, dydx = e.$$

$$(21)\quad \int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{2}{(1+x^2+y^2)^2} dydx = \int_0^1 \int_0^{2\pi} \frac{2}{(1+r^2)^2} d\theta dr.$$

$$(22)\quad \int_0^2 \int_0^{4-x^2} \int_0^x \frac{\sin 2z}{4-z} dydzdx = \int_0^4 \int_0^2 \int_y^{\sqrt{4-z}} \frac{\sin 2z}{4-z} dxdydz$$

(23) A solid region in the first octant is bounded by the coordinate planes and the plane $x + y + z = 2$. The density of the solid is $\delta(x, y, z) = 2x$. The mass of the solid is $4/3$.

(24) The volume of the solid bounded below by the sphere $\rho = 2\cos\phi$ and above by the cone $z = \sqrt{x^2 + y^2}$ is $\int_0^{2\pi} \int_0^{\pi/4} \int_0^{2\cos\phi} \rho^2 \sin\phi d\rho d\phi d\theta$.

(25) Along the curve $C: \ \mathbf{r}(t) = 4\cos t\mathbf{i} + 4\sin t\mathbf{j} + 3t\mathbf{k}$, $-2\pi \le t \le 2\pi$, $\int_C \sqrt{x^2 + y^2} ds = 80\pi$

(26) For the vector field $\mathbf{F} = x^2\mathbf{i} - y\mathbf{j}$ along the curve $C: \ x = y^2$ from $(4, 2)$ to $(1, -1)$, $\int_C \mathbf{F} \cdot \mathbf{T} ds = \int_2^{-1} (2y^5 + y) dy$

(27) Let $C$ be the line segment from $(1, 1, 1)$ to $(1, 2, 1)$, and to $(-1, 1, 0)$. Then

$$\int_C e^x \ln y dx + (\frac{e^x}{y} + \sin z) dy + y \cos z dz = \sin 1$$

(28) Let $C$ be the boundary of the region $0 \le x \le \pi$, $0 \le y \le \sin x$ in counterclockwise direction. The $\int_C 3y dx + 2x dy = -2$

(29) Let $C$ be the boundary of a polygonal domain $D$ in counterclockwise direction. Then the area of $D$ is equal to $\int_C (-y) dx$.

(30) Let $C$ be a simple closed smooth curve in the plane $2x + 2y + z = 2$. Then $\int_C 2y dx + 3z dy - x dz = \pm A$, where $A$ is the area of the region enclosed by $C$.

(31) The outward flux of the position vector field $\mathbf{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ through a smooth closed surface $S$ is three thimes the volume of the region enclosed by the surface.

(32) The point closest to the origin on the curve of intersection of the plane $2y + 4z = 5$ and the cone $z^2 = 4x^2 + 4y^2$ must satisfy the equations $\bigtriangledown(z^2 - (4x^2 + 4y^2)) = \lambda \bigtriangledown (2y + 4z - 5)$ for some scalar $\lambda$.

**For problems (33)-(35), please write down your procedure as much as possible.** (Two points for each).

(33) Determine whether the function $f(x, y) = x^4y^2/(x^6 + y^4)$ has limit or not as $(x, y) \to (0, 0)$.

(34) Let

$$f(x,y) = \begin{cases} xy\frac{x^2-y^2}{x^2+y^2}, & \text{if } (x,y) \neq 0, \\ 0, & \text{if } (x,y) = 0. \end{cases}$$

Find $\frac{\partial^2 f}{\partial y \partial x}(0, 0)$.

(35) Find all the local maxima, local minima and saddle points of the function $f(x, y) = \ln(x + y) + x^2 - y$.