Homework 2

1. For $u_t + a u_x = 0$ ("a" is a constant), derive an implicit Lax-Wendroff scheme and study its stability.

2. Verify that Strang spliting is 2nd order accurate (i.e., it has 3rd order local error) for linear systems of hyperbolic equations $\vec{u}_t + A\vec{u}_x + B\vec{u}_y = 0$, where A and B are constant square matrices.

3. $$\begin{cases} u_t + u_x = 0, \quad t > 0, \quad x \in (0,1). \\ u(x,0) = -\sin(3\pi x), \text{ if } x \in [0, \tfrac{1}{3}); \; 1 \text{ if } x \in [\tfrac{1}{3}, \tfrac{2}{3}); \\ \qquad\qquad 0 \text{ if } x \in [\tfrac{2}{3}, 1]. \\ u(0,t) = u(1,t), \quad t > 0. \end{cases}$$

Compute the solution at $t = 10$ with at least 6 methods. Plot your numerical solutions against the exact solution at $t = 10$.

4. Derive a Peaceman–Rachford scheme for the non homogeneous equation $u_t = \Delta u + f(x, y, t)$, where f is a smooth function. Study its truncation error and stability.